

THE PRODUCT RULE

Math 130 - Essentials of Calculus

5 March 2021

EXPANDING OUR ABILITIES

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THEOREM (THE PRODUCT RULE)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

EXAMPLES

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Differentiate the following functions

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③ $R(t) = (t + e^t)(3 - \sqrt{t})$

④ $k(x) = \sin x \cos x$

MORE EXAMPLES

EXAMPLE

Find the derivative of $f(x) = x^2 e^x \cos x$.

MORE EXAMPLES

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Find the derivative of $f(x) = x^2 e^x \cos x$.

EXAMPLE

Find the second derivative of $f(x) = x^3 \sin x$.

FUN FACTS

PROPERTY (TRIPLE PRODUCT RULE)

If f , g , and h are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

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PROPERTY (2ND PRODUCT RULE)

If f and g are twice differentiable functions, then

$$\frac{d^2}{dx^2}[f(x)g(x)] = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x).$$